B. Sc. Part II 4th Paper

DIFF. EQNS. (contd)

LDECC

Q. Solve

$$(D^4 + 6D^3 + 11D^2 + 6D)y = 20\, e^{-2x} \sin x$$

Soln. For CF, $D^4 + 6D^3 + 11D^2 + 6D = 0$

$$\Rightarrow D(D^3 + 6D^2 + 11D + 6) = 0$$

$$\Rightarrow D[D^3 + D^2 + 5D^2 + 5D + 6D + 6] = 0$$

$$\Rightarrow D[D^2(D+1) + 5D(D+1) + 6(D+1)] = 0$$

$$\Rightarrow D(D+1)(D^2 + 5D + 6) = 0$$

$$\Rightarrow D(D+1)(D+3)(D+2) = 0 \Rightarrow D = 0, -1, -2, -3$$

$$\therefore CF = C_1 + C_2 e^{-x} + C_3 e^{-2x} + C_4 e^{-3x}$$

Now, $$PI = \frac{1}{D^4 + 6D^3 + 11D^2 + 6D} 20\, e^{-2x} \sin x$$

$$= \frac{20}{D(D^3 + 6D^2 + 11D + 6)} e^{-2x} \cdot \sin x$$

$$= 20 \cdot e^{-2x} \frac{1}{D(D+1)(D+2)(D+3)} \sin x$$

$$= 20\, e^{-2x} \frac{1}{(D-2)(D-2+1)(D-2+2)(D-2+3)} \sin x$$

$$\Rightarrow PI = 20e^{-2x} \frac{1}{D(D-1)(D-2)(D+1)} \sin x$$

$$\text{or } PI = 20e^{-2x} \frac{1}{(D^2-2D)(D^2-1)} \sin x$$

$$\text{or } PI = 20e^{-2x} \frac{1}{(-1^2-2D)(-1^2-1)} \sin x$$

$$\text{or } PI = 20e^{-2x} \frac{1}{+2(1+2D)} \sin x$$

$$\text{or, } PI = 10e^{-2x} \frac{1}{(1+2D)} \xrightarrow{(1-2D)} \xrightarrow{(1-2D)} \sin x$$

$$\text{or, } PI = 10e^{-2x} \frac{1-2D}{(1+2D)(1-2D)} \sin x$$

$$\text{or } PI = 10e^{-2x} \frac{1-2D}{1-4D^2} \sin x$$

$$\text{or } PI = 10e^{-2x} \frac{(1-2D)\sin x}{1-4(-1^2)}$$

$$\text{or, } PI = \frac{10e^{-2x}[\sin x - 2D(\sin x)]}{1+4}$$

$$\Rightarrow PI = 2e^{-2x}(\sin x - 2\cos x)$$

Hence, the soln is given by $y = CF + PI$

$$\Rightarrow y = c_1 + c_2 e^{-x} + c_3 e^{-2x} + c_4 e^{3x} + 2e^{-2x}(\sin x - 2\cos x)$$